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Quantifying Anomalous Coastal Processes with Fractional Calculus-Based Models

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The emerging mathematical methods developed from fractional calculus are worth immediate attention in marine and coastal sciences.

The term "anomalous diffusion" refers to the mechanisms described by a fractional differential equation (FDE) or fractional partial differential equation (FPDE) compared to the processes described by the classic diffusion equation which is regarded as the "normal" diffusion [\[10](#page-1-0), [16\].](#page-1-1) The recent advances in many fields have demonstrated that the anomalous process is universal, which implies that coastal processes are also anomalous.

The oceans are vital to human societies, life and the environment on the planet in many aspects by creating continuous hydrological cycles to provide water for sustaining life, environmental health and activities ensuring life's survival. The United Nations Environment Programme (UNEP) [\[12\]](#page-1-2) estimates that "*Half the world's population lives within 60 km of the sea, and three-quarters of all large cities are located on the coast. However, the seas and oceans are under increasing pressure from pollution. Much of this pollution comes from urban centres, and it creates environmental problems which threaten the viability of the cities themselves*." According to the United Nations Development Programme (UNDP) [\[11\]](#page-1-3), "*Globally, the market value of marine and coastal resources and industries is estimated at \$3 trillion per year or about 5% of global GDP, and an estimated 63% of global 'ecosystems services' are provided by marine and coastal systems. As much as 40% of the world oceans are considered as 'heavily affected' by human activities, including pollution, depleted fisheries, loss of coastal habitats such as coral reefs, mangroves and seagrasses, and by aquatic invasive species*."

The oceans of the world function as an irreplaceable source for water to replenish the otherwise dry and dead continents while embracing the influxes of various forms of pollutants, contaminants, pathogens, radioactive and other destructive materials. The data from the UN Water Assessment Programme [\[13\]](#page-1-4) (2009, p. 138) indicates that "*In at least 8 of the United Nations Environment Programme's 13 Regional Seas Programme regions, over 50% of the wastewater discharged into freshwater and coastal areas is untreated, rising to over 80% in 5 regions*." These facts depict a grim situation for the health of the oceans.

The importance of the oceans and coastal regions to humans, the existing problems and challenges in coastal regions require consistent efforts and remedies to reduce the burdens on oceans and coastal regions, which require knowledge and tools for the quantification of water and solute exchanges between fresh and sea waters. With the inclusion of rainfall, physical evaporation and transpiration by plants, quantification of the exchanges between seawater and freshwater and solutes entrained within them pose challenges for numerical simulation and measurements for model parameters. While some of the existing models, particularly some numerical codes such as SUTRA [\[14\]](#page-1-5) enjoyed success, our understanding of the physical, chemical, biological and other processes along coastal regions with limited data and simulated results is insufficient to inform various kinds of users such as irrigators on coastal farms, marine engineers, coastal conservationists and coastal region managers *etc*.

Recent advances in applying fractional calculus are encouraging [\[3,](#page-1-6) [4](#page-1-7), [9\]](#page-1-8). In hydrology, very generic fractional partial differential equations (FPDEs) for solute movement in aquifers [\[2](#page-1-9), [16\],](#page-1-1) water movement in soils [\[6,](#page-1-10) [7\]](#page-1-11) and groundwater flow in aquifers [\[8\]](#page-1-12) have also been presented. Now the question is how the interactions between seawater and freshwaters in the coastal regions can be modelled using the FPDEs in order for the mathematical analyses across the different fields to be compatible. The fractional

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Editorial

calculus-based models and their parameters provide much more information about the stochastic processes of water flow and solute movement in porous media and fluid mechanics in the framework of the continuous-time random walk (CRTW) theory, which also connects the fractal geometry and the FPDEs [\[15\]](#page-1-13).

Modelling coastal hydrological processes using FPDEs will certainly require more effort for parameterising the models compared to that for models based on integer calculus and integer partial differential equations (PDEs) such as in Bear (2005) [\[1\],](#page-1-14) Liu et al. (2006) [\[5\]](#page-1-15), and Voss and Provost (2010) [\[14\]](#page-1-5). The emergency of FPDEs in hydrological modelling is an important issue for consideration in that the extra information in the FPDE-based techniques underscores the advantages over their traditional counterparts. The processes, which can be modelled using these FPDE-based approaches, include coastal and marine processes and phenomena such as ocean waves, near-shore tidal waves, beach erosion processes, water movement and solute (salts and microbes etc.) transport in soils and aquifers, surface water and groundwater interactions, exchange between fresh water and seawater and biogeochemical cycles etc.

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