International Journal of Aeronautical Science & Aerospace Research (IJASAR)
ISSN: 2470-4415

UAV Flight Dynamics

Review Article

Philip Scott Blackwelder

Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, USA.

Abstract

In the interest of promoting the integration of hybrid-electric power train into the aviation industry, research is being conducted by North Carolina State University to establish the feasibility of electrified power train in a small scale unmanned aerial vehicle (UAV). To accomplish this, it is first necessary to understand dynamics of the system to calculate the required power associated with each portions of the aircraft's mission. Though research that has been conducted in the past based power required on published governing equations. However, in the interest of understand the system at its most fundamental level, it was deemed prudent to derive the governing equations from Newton's second law.

Using analytical dynamics, a rotating coordinate system was applied to the craft and a rotation matrix was applied to establish the interaction of the aircraft's external forces based on its Euler angles with respect to the inertial frame.

The equations associated with the values and interactions of the aircraft's external forces were combined and manipulated to develop three governing parameters of maintaining flight during a typical UAV mission. These parameters are minimum airspeed, minimum power applied to the propeller, and the minimum roll angle required to accomplish a loitering maneuver with set radius about a target.

The next step of this research will be development of an optimization algorithm to match power train components to apply the necessary conditions derived in this paper.

Keywords: Unmanned Ariel Vehicle; UAV; Flight Dynamics; Analytical Dynamics.

Glossary of Variables:

k_T	– Thrust coefficient of the propeller
ϱ	– Air density
n	– Rotational speed of the propeller (rev/s)
D	– Propeller's Diameter (in)
A_p	– Wing planform area
C_L	– Wing coefficient of lift
J	– Propeller advance ratio (u/nD)
A_F	– Aircraft frontal area
C_D	– Sum of the parasitic (C_{D_0}) and lift induced (C_{D_i}) drag coefficients
AR	– Wing aspect ratio
e	– Oswald efficiency factor

***Corresponding Author**:
Philip Scott Blackwelder,
Department of Mechanical and Aerospace Engineering, North Carolina State University Raleigh, 27695, USA.
E-mail: psblackw@ncsu.edu

Received: November 11, 2015
Accepted: November 13, 2015
Published: December 21, 2015

Citation: Philip Scott Blackwelder (2015) UAV Flight Dynamics. *Int J Aeronautics Aerospace Res.* 2(6), 81-85. **doi: http://dx.doi.org/10.19070/2470-4415-150009**

Introduction

Though most associate unmanned aerial vehicles (UAVs) with the military, the number of nonmilitant applications is growing everyday including precision agriculture, package delivery, geological survey, and a myriad of other public applications. As such, it is becoming increasingly important for these vehicles to not only support longer mission durations but to also do so in a way that is cost effective and minimally pollutant both in emissions and noise. Given the current state of battery technology, purely electric UAVs fail to provide desirable mission durations. In addition, the noise and pollution associated with gasoline or other internal combustion engine powered UAVs is undesirable for applications in populated areas. Given these constraints, the best solution to achieve the desired goals of clean, quiet, and extended missions is to develop a hybrid-electric UAV platform. Hybrid-electric power train leverages the advantages of both

energy sources while minimizing the disadvantages. This paper will outline the dynamics associated with a typical UAV missions and offer governing equations from which a hybrid-electric power train platform can be designed and optimized to offer peak efficiency while minimizing noise and emissions.

Power Required to Maintain Flight

Newton's second law, the sum of the external forces on the aircraft is equal to the product of the craft's mass and the acceleration of its center of mass with respect to the inertial frame (Figure 1).

$$\sum \vec{F}_{external} = m * \bar{\vec{a}}_{cm/o}$$

Define:

$$\vec{rcm}_{/B} = L_{sm}\vec{i}_{\bar{B}}$$

$$\vec{v}_{B/o} = u\vec{i}_{\bar{B}} + v\vec{j}_{\bar{B}} + w\vec{k}_{\bar{B}}$$

$$\bar{\vec{\omega}}^{\bar{o}\to\bar{B}} = p\vec{i}_{\bar{B}} + q\vec{j}_{\bar{B}} + r\vec{k}_{\bar{B}}$$

Derivation of Aircraft's Acceleration Vector in the B Frame

$$\bar{\vec{a}}_{cm/o} = \frac{\bar{o}d^2}{dt^2}\vec{r}_{cm/o} = \frac{\bar{o}d^2}{dt^2}\vec{r}_{cm/B} + \frac{\bar{o}d^2}{dt^2}\vec{r}_{B/o} = \frac{\bar{o}d^2}{dt^2}\vec{r}_{cm/B} + \frac{\bar{o}d}{dt}\vec{v}_{B/o}$$

$$\frac{\bar{o}d}{dt}\vec{v}_{B/o} = \frac{\bar{B}d}{dt}\vec{v}_{B/o} + \bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times \vec{v}_{B/o}$$

$$= \dot{u}\vec{i}_{\bar{B}} + \dot{v}\vec{j}_{\bar{B}} + \dot{w}\vec{k}_{\bar{B}} + (p\vec{i}_{\bar{B}} + q\vec{j}_{\bar{B}} + r\vec{k}_{\bar{B}}) \times (u\vec{i}_{\bar{B}} + v\vec{j}_{\bar{B}} + w\vec{k}_{\bar{B}})$$

$$= \dot{u}\vec{i}_{\bar{B}} + \dot{v}\vec{j}_{\bar{B}} + \dot{w}\vec{k}_{\bar{B}} + [(qw - rv)\vec{i}_{\bar{B}} + (ru - pw)\vec{j}_{\bar{B}} + (pv - qu)\vec{k}_{\bar{B}}]$$

$$= (\dot{u} + qw - rv)\vec{i}_{\bar{B}} + (\dot{v} + ru - pw)\vec{j}_{\bar{B}} + (\dot{w} + pv - qu)\vec{k}_{\bar{B}}$$

$$\frac{\bar{o}d^2}{dt^2}\vec{r}_{cm/B} = \frac{\bar{B}d^2}{dt^2}\vec{r}_{cm/B} + (\bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times \frac{\bar{B}d}{dt}\vec{r}_{cm/B}) + (\bar{\vec{\alpha}}^{\bar{o}\to\bar{B}} \times \vec{r}_{cm/B}) + \bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times (\bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times \vec{r}_{cm/B})$$

$$\frac{\bar{B}d^2}{dt^2}\vec{r}_{cm/B} = \frac{\bar{B}d^2}{dt^2}L_{sm}\vec{i}_B = 0$$

$$\frac{\bar{B}d}{dt}\vec{r}_{cm/B} = \frac{\bar{B}d^2}{dt^2}L_{sm}\vec{i}_B = 0$$

$$\bar{\vec{\alpha}}^{\bar{o}\to\bar{B}} = \frac{\bar{B}d}{dt}(\bar{\vec{\alpha}}^{\bar{o}\to\bar{B}}) = \frac{\bar{B}d}{dt}(u\vec{i}_{\bar{B}} + v\vec{j}_{\bar{B}} + w\vec{k}_{\bar{B}}) = \dot{p}\vec{i}_{\bar{B}} + \dot{q}\vec{j}_{\bar{B}} + \dot{r}\vec{k}_{\bar{B}}$$

$$\therefore \frac{\bar{o}d^2}{dt^2}\vec{r}_{cm/B} = \left(\bar{\vec{\alpha}}^{\bar{o}\to\bar{B}} \times \vec{r}_{cm/B}\right) + \left[\bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times \left(\bar{\vec{\omega}}^{\bar{o}\to\bar{B}} \times \vec{r}_{cm/B}\right)\right]$$

$$= [(\dot{p}\vec{i}_{\bar{B}} + \dot{q}\vec{j}_{\bar{B}} + \dot{r}\vec{k}_{\bar{B}}) \times (L_{sm}\vec{i}_{\bar{B}})] + [(p\vec{i}_{\bar{B}} + q\vec{j}_{\bar{B}} + r\vec{k}_{\bar{B}}) \times ((p\vec{i}_{\bar{B}} + q\vec{j}_{\bar{B}} + r\vec{k}_{\bar{B}}) \times (L_{sm}\vec{i}_{\bar{B}}))]$$

$$= [-\dot{q}L_{sm}\vec{k}_{\bar{B}} + \dot{r}L_{sm}\vec{j}_{\bar{B}}] + [(p\vec{i}_{\bar{B}} + q\vec{j}_{\bar{B}} + r\vec{k}_{\bar{B}}) \times (-qL_{sm}\vec{k}_{\bar{B}} + rL_{sm}\vec{j}_{\bar{B}})]$$

$$= [-\dot{q}L_{sm}\vec{k}_{\bar{B}} + \dot{r}L_{sm}\vec{j}_{\bar{B}}] + [-L_{sm}(r^2 + q^2)\vec{i}_{\bar{B}} + pqL_{sm}\vec{j}_{\bar{B}} + prL_{sm}\vec{k}_{\bar{B}}]$$

$$= -L_{sm}(r^2 + q^2)\vec{i}_{\bar{B}} + L_{sm}(pq + \dot{r})\vec{j}_{\bar{B}} + L_{sm}(pr - \dot{q})\vec{k}_{\bar{B}}$$

$$\therefore \bar{\vec{a}}_{cm/o} = [(\dot{u} + qw - rv)\vec{i}_{\bar{B}} + (\dot{v} + ru - pw)\vec{j}_{\bar{B}} + (\dot{w} + pv - qu)\vec{k}_{\bar{B}}] +$$
$$[-L_{sm}(r^2 + q^2)\vec{i}_{\bar{B}} + L_{sm}(pq + \dot{r})\vec{j}_{\bar{B}} + L_{sm}(pr + \dot{q})\vec{k}_{\bar{B}}]$$

$$= (\dot{u} + qw - rv - L_{sm}(r^2 + q^2))\vec{i}_{\bar{B}} + (\dot{v} + ru - pw + L_{sm}(\dot{p}q + \dot{r}))\vec{j}_{\bar{B}}$$
$$+ (\dot{w} + pv - qu + L_{sm}(pr - \dot{q}))\vec{k}_{\bar{B}}$$

External Forces Acting Upon the Aircraft

From the free body diagram:

$$\sum \vec{F}_{external} = (F_T - F_D)\vec{i}_{\bar{B}} - F_c\vec{j}_{\bar{o}} - F_L\vec{k}_{\bar{B}} + W\vec{k}_{\bar{o}}$$

Equation 1 - Sum of External Forces Acting Upon the Aircraft

The term $mg\vec{k}_{\bar{o}}$ must be rotated into the $\bar{B}$ frame; therefore a rotation matrix must be applied. The rotation about the inertial frame can be expressed with Euler angles. The vehicle will rotate Φ about $\vec{i}_{\bar{o}}$, θ about $\vec{j}_{\bar{o}}$, and ψ about $\vec{k}_{\bar{o}}$.

$$[R_\theta]_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad [R_\phi]_y = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$[R_\psi]_z = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore:

$$\begin{bmatrix} \vec{i}_{\bar{B}} \\ \vec{j}_{\bar{B}} \\ \vec{k}_{\bar{B}} \end{bmatrix} = [R_\theta]_x[R_\phi]_x[R_\psi]_x \begin{bmatrix} \vec{i}_{\bar{o}} \\ \vec{j}_{\bar{o}} \\ \vec{k}_{\bar{o}} \end{bmatrix}$$

$$\begin{bmatrix} \cos\phi\cos\varphi & \cos\phi\sin\varphi & -\sin\phi \\ \cos\varphi\sin\phi\sin\theta - \cos\theta\sin\varphi & \cos\varphi\cos\theta - \sin\phi\sin\varphi\sin\theta & \cos\phi\sin\theta \\ \sin\varphi\sin\theta + \cos\varphi\cos\theta\sin\phi & \cos\theta\sin\phi\sin\varphi - \cos\varphi\sin\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \vec{i}_{\bar{o}} \\ \vec{j}_{\bar{o}} \\ \vec{k}_{\bar{o}} \end{bmatrix}$$

Therefore the forces in Equation 1 referencing the inertial frame can be expressed with respect to the aircraft's frame:

$$F_C = \{\frac{mu^2}{R}\cos\varphi\sin\phi\sin\theta - \cos\theta\sin\varphi\frac{mu^2}{R}\cos\varphi\cos\theta +$$
$$\sin\phi\sin\varphi\sin\theta\frac{mu^2}{R}\cos\phi\sin\theta\}_{\bar{B}}$$

$$W = \{-\sin\phi\cos\phi\sin\theta\cos\phi\cos\theta\}_{\bar{B}}$$

Assuming the angle $\varphi = 0$ at steady state, the sum of external forces then becomes:

Figure 1. Inertial and Rotating Reference Frames.

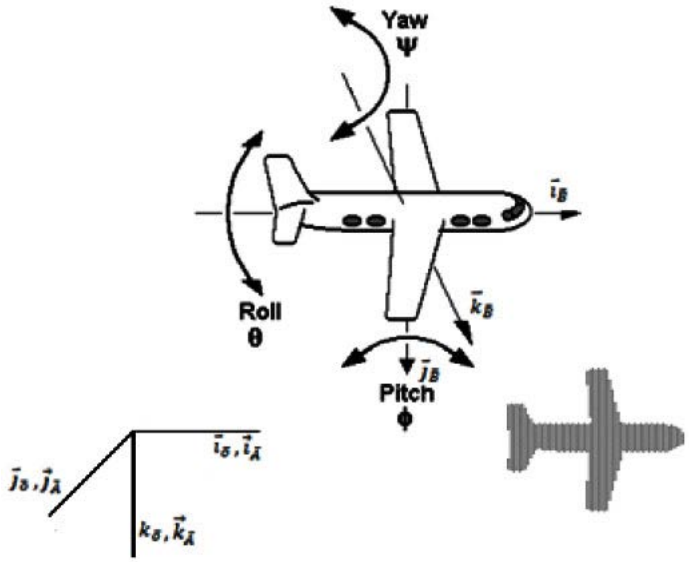


Figure 2. Free Body Diagram of the Aircraft during Level Flight.

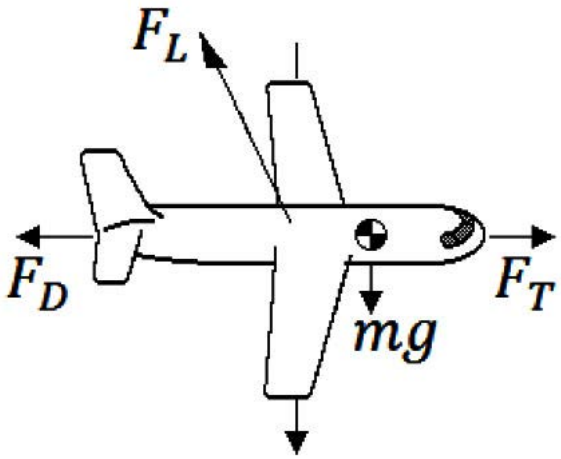
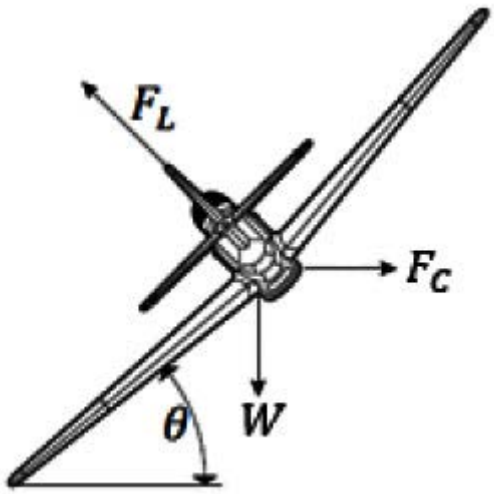


Figure 3. Free Body Diagram of the Aircraft during a Banking Maneuver.



$$\sum \vec{F}_{external} = \left[F_T - F_D + \frac{mu^2}{R} \sin\phi \sin\theta - mg \sin\phi \right] \vec{i}_{\vec{B}} +$$

$$\left[mg(\cos\phi \sin\theta) - \frac{mu^2}{R} \cos\theta \right] \vec{j}_{\vec{B}} + \left[mg(\cos\phi \cos\theta) - F_L - \frac{mu^2}{R} \cos\phi \sin\theta \right] \vec{k}_{\vec{B}}$$

Setting the above equation equal to the expression found using Newton's second law yields:

$$F_T - F_D + \frac{mu^2}{R} \sin\phi \sin\theta - mg \sin\phi = m[\dot{u} + qw - rv - L_{sm}(r^2 + q^2)]$$

$$mg(\cos\phi \sin\theta) - \frac{mu^2}{R} \cos\theta = m[\dot{v} + ru - pw + L_{sm}(\dot{p}q + \dot{r})]$$

$$mg(\cos\phi \cos\theta) - F_L - \frac{mu^2}{R} \cos\phi \sin\theta = m[\dot{w} + pv - qu + L_{sm}(pr + \dot{q})]$$

Therefore for the aircraft to maintain flight, the following must remain true:

$$F_T \geq F_D - \frac{mu^2}{R} \sin\phi \sin\theta + mg \sin\phi + m[\dot{u} + qw - rv - L_{sm}(r^2 + q^2)]$$

Equation 2 – Thrust Force Required to Sustain Flight

$$F_L \geq mg(\cos\phi \cos\theta) - \frac{mu^2}{R} \cos\phi \sin\theta - m[\dot{w} + pv - qu + L_{sm}(pr - \dot{q})]$$

Equation 3 – Lift Force Required to Sustain Flight

The roll angle required to maintain equilibrium (assuming Φ=0) during loitering can also be derived from the sum of the forces about the j axis:

$$mg\,(\cos 0 \sin\theta) = (mu^2/R)\cos\theta$$

$$\therefore \frac{u^2}{R} = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \theta = \tan^{-1} \frac{u^2}{Rg}$$

Equation 4 - Roll Angle Required During Loitering

External Forces

Thrust Force

The thrust force (F_T) is expressed as:

$$F_T = k_T \rho n^2 D^4$$

Equation 5 - Thrust Force Equation [1]

To establish the operating point of the engine to provide adequate thrust for maintaining flight, some relations can be applied using information about the propeller.

The propeller's torque coefficient can be expressed as:

$$k_Q = \frac{Q}{\rho n^2 D^5}$$

Equation 6 - Propeller Torque Coefficient [1]

The torque coefficient can be related to the thrust coefficient through the propeller's efficiency:

$$\eta_P = \frac{k_T}{2\pi k_Q} J$$

Equation 7 - Propeller Efficiency with Respect to Advance Ratio [1]

Rearranging Equation 7:

$$k_T = \frac{\eta_P 2\pi k_Q}{J} = \frac{\eta_P 2\pi k_Q nD}{u}$$

Equation 8 - Thrust Coefficient with Respect to Prop Efficiency and Torque Coefficient

Plugging in Equation 6:

$$k_T = \frac{\eta_P 2\pi QnD}{u\rho n^2 D^5} = \frac{\eta_P 2\pi Q}{u\rho nD^4}$$

Equation 9 - Thrust Coefficient with Respect to Torque and Angular Velocity

Plugging Equation 9 into Equation 5:

$$F_T = \left(\frac{\eta_P 2\pi Q}{\eta \rho nD^4} \right) \rho n^2 D^4 = \frac{\eta_P 2\pi nQ}{u}$$

Lift Force

The lift force (F_L) is expresses as:

$$F_L = \frac{1}{2} \rho A_P u^2 C_L$$

Equation 10 - Lift Force

Drag Force

The total drag force (F_D) is expressed as:

$$F_D = \frac{1}{2} \rho A_F u^2 C_D$$

The coefficient of drag C_D, is the sum of the coefficient of parasitic drag C_{D_0} and the lift induced drag coefficient. The lift induced drag coefficient is expressed as:

$$C_{Di} = \frac{C_L^{\,2}}{\pi ARe}$$

Therefore, the total drag is expressed as:

$$F_D = \frac{1}{2}\rho A_F u^2 \left(C_{Do} + \frac{C_L^{\,2}}{\pi ARe} \right)$$

Equation 11 - Total Drag Force

Plugging Equation 11 into Equation 2 and solving for Qn yields:

$$Qn \geq \frac{u}{\eta_P 2\pi} \left[\begin{array}{l} \frac{1}{2}\rho A_F u^2 \left(C_{Do} + \frac{C_L^{\,2}}{\pi ARe} \right) - \frac{mu2}{R}\sin\phi\sin\theta + \\ mg\sin\phi + m[\dot{u} + qw - rv - L_{sm}(r^2 + q^2)] \end{array} \right]$$

Equation 12 – Input Torque/Speed Required to Surmount Drag

Plugging Equation 10 into Equation 3 yields:

$$u \geq \sqrt{\frac{2}{\rho A_P C_L}} \left[\begin{array}{l} mg(\cos\phi\cos\theta) - \frac{mu^2}{R}\cos\phi\sin\theta - \\ m[\dot{w} + pv - qu + L_{sm}(pr - \dot{q})] \end{array} \right]$$

Equation 13 – Airspeed Required to Surmount Weight

Equation 4, Equation 12, and Equation 13 serve as necessary conditions to sustain flight over the course of the mission. Equation 12 dictates the minimum combination of torque and shaft speed that must be delivered to the propeller, Equation 13 dictates the minimum speed that must be maintained to prevent stall, and Equation 4 dictates the minimum roll angle required to maintain a constant loitering radius about the target at the desired air speed.

Conclusion

The rapidly expanding market for UAVs will bring with it demand for more flexible platforms that can be quickly and easily assembled and modified based on mission parameters. Additionally, the increasing precedence of reducing fuel consumption and emissions is forcing designers to think more and more outside of the box to satisfy radically conflicting design requirements. Though adding additional complexity to an already sophisticated system can seem arduous, a firm understanding of the fundamental governing equations and the relationship between the various inputs and outputs, the design process becomes much more seamless.

Acknowledgements

I would like to thank Dr. Eric Klang for his patience and guidance over the ten years we have worked together at NC State University. I would also like to thank Dr. James Kribs for his selfless devotion to education in addition to facilitating and supervising all aerodynamic testing and for being my friend.

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